Online Workshop on Modeling and Numerical Methods for Interface Problems

Conference Brochure

Shenzhen, China

Jan 28-30, 2021

Southern University of Science and Technology, China

中国南方科技大学

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SUSTech Southern University of Science and Technology

1. Information

- **Date** Jan 28-30, 2021
- **Venue** ZOOM ID **669 417 9735**
- **Organizing Committee**

  Prof. WANG Xiaoming (SUSTech)

  Prof. WANG Xiao-Ping (HKUST)

  Prof. YANG Jiang (SUSTech)

  Prof. ZHANG Zhen (SUSTech)

- **Secretary**

  GUO Yichen (SUSTech) guoyc@mail.sustech.edu.cn

2. Conference Schedule

Jan 28, 2021 (Thursday)			
8:30--9:10	SHEN Jie	Highly efficient and accurate positivity preserving and energy stable schemes for second- and fourth-order nonlinear systems	Chair: WANG Xiaoming
9:10--9:50	HAN Daozhi	A superconvergent hybridizable discontinuous Galerkin method for solving the Cahn-Hilliard equation	
9:50--10:00	Break		
10:00--10:40	ZHANG Qinghai	Fourth-order projection methods for incompressible Navier-Stokes equations on irregular no-slip domains	Chair: BAO Weizhu
10:40--11:20	ZHENG Weiying	A high-order fictitious-domain method for linear convection-diffusion equation on time-varing domain	
11:20-12:00	WEI Chaozhen	Modeling growth and morphogenesis in biological tissues	
12:00--15:30	Break		
15:30-16:10	XIANG Yang	Modeling Effects of Randomness on the Strength of High-Entropy Alloys	Chair: REN Weiqing
16:10--16:50	DAI Shuyang	Multiscle modeling of grain boundaries based on Peierls Nabarro description	
16:50--17:30	ZHANG Luchan	Dynamics of Grain Boundaries with Microscopic Constraints	
18:10--20:30	Break		
20:30--21:10	REN Weiqing	Simulation of electro-wetting on dielectric	Chair: WANG Xiao-Ping
21:10-21:50	ZHAO Quan	A finite element method for Electrowetting on Dielectric	

Jan 29, 2021 (Friday)			
8:30--9:10	LI Shuwang	Modeling and nonlinear simulation of solid tumor growth with chemotaxis	Chair: LI Zhilin
9:10--9:50	YING Wenjun	A cell-centered finite volume method on locally refined composite Cartesian grids and its application in wetting problems	
9:50--10:00	Break		
10:00--10:40	LIU Chun	Boundary Effects in General Diffusion	Chair: LAI Ming-Chih
10:40--11:20	WANG Yiwei	On variational Lagrangian methods for phase-field models and beyond: A discrete energetic variational approach	
11:20-12:00	FENG Xinlong	Local tangential lifting method for moving interface problems on surfaces with applications	
12:00--14:30	Break		
15:30--16:10	SUN Shuyu	Modeling Multi-Component Two-Phase Systems with Partial Miscibility using Diffuse Interface Models Based on Equations of State	Chair: YANG Jiang
16:10--16:50	XU Xianmin	A diffusion-generated method for wetting on inhomogeneous surfaces	
16:50-17:30	XU Xinpeng	Onsager's variational principles in active soft matter	Chair: ZHANG Zhen
17:30-18:10	WANG Dong	An efficient iterative convolution thresholding method (ICTM) for free interface related optimization problems	
18:10—20:30	Break		
20:30--21:10	LIN Ping	Thermodynamically consistent phase-field modelling and its application in moving contact line problems and vesicle motions	Chair: QIAN Tiezheng
21:10--21:50	XU Shixin	An Energy Stable C^0 Finite Element Scheme for A Phase-Field Model of Vesicle Motion and Deformation	

Jan 30, 2021 (Saturday)			
8:30--9:10	WANG Qi	Thermodynamically consistent algorithms for incompressible diblock copolymer solutions coupled with electric and magnetic fields	Chair: SHEN Jie
9:10--9:50	YUE Pengtao	Phase-field modeling of contact angle hysteresis and its application in drop impact dynamics	
9:50--10:00	Break		
10:00--10:40	BAO Weizhu	Energy-stable parametric finite element methods (PFEM) for geometric PDEs and applications	Chair: LIU Chun
10:40--11:20	JIANG Wei	Sharp-interface approaches for simulating solid-state dewetting problems	

3. Titles & Abstracts

Energy-stable parametric finite element methods (PFEM) for geometric PDEs and applications

BAO Weizhu
National University of Singapore

In this talk, I begin with a review of different geometric flows (PDEs) including mean curvature (curve shortening) flow, surface diffusion flow, Willmore flow, etc., which arise from materials science, interface dynamics in multi-phase flows, biology membrane, computer graphics, geometry, etc. Different mathematical formulations and numerical methods for mean curvature flow are then discussed. In particular, an energy-stable semi-implicit parametric finite element method (PFEM) is presented in details. Then the PFEM is extended to surface diffusion flow and anistropic surface diffusion flow. Finally, sharp interface models and their PFEM approximations are presented for solid-state dewetting. This talk is based on joint works with Wei Jiang, Yifei Li, Yan Wang and Quan Zhao.

Multiscle modeling of grain boundaries based on Peierls Nabarro description

DAI Shuyang
Wuhan University

We develop a multiscale continuum model to describe the interface structure in crystalline material such as FCC metals. The interface structure for twist, tilt and misfit grain boundaries are described by the dislocation network. The model incorporates both the anisotropy elasticity of each grain in crystalline materials and the molecular dynamics calculation informed interaction between two bulks, i.e., the nonlinear generalized stacking-fault energy. The equilibrium structures are obtained from the numerical simulations of the force balance differential equations. We apply this approach to determine the structure and energetics of twist, tilt and general grain boundaries. We also investigated the dislocation structure in heterogeneous crystalline material. Our model agrees well with the atomistic results. An analytical description is developed based on the obtained structural features.

Local tangential lifting method for moving interface problems on surfaces with applications

FENG Xinlong
Xinjiang University

In this work, a new numerical computational frame is presented for solving moving interface problems modeled by parabolic PDEs on static and evolving surfaces. The surface PDEs can have a Dirac delta source distributions and discontinuous coefficients. One application is for thermally driven moving interfaces on surfaces such as Stefan problems and dendritic solidification phenomena on solid surfaces. Novelties of the proposed method is the local tangential lifting method to construct discrete delta functions on surfaces. The idea of the local tangential lifting method is to transform a local surface problem to a local 2D problem on the tangent planes of surfaces. A surface version of the front tracking method is developed to track moving interfaces on surfaces. Strategies have been developed for computing geodesic curvatures of interfaces on surfaces. Various numerical examples are presented to demonstrate the accuracy and reliability of the proposed method.

A superconvergent hybridizable discontinuous Galerkin method for solving the Cahn-Hilliard equation

HAN Daozhi
Missouri University of Science and Technology

In this talk we introduce a superconvergent hybridizable discontinuous Galerkin method for solving the Cahn-Hilliard equation. The temporal discretization can be either based on either backward Euler method or convex-splitting. We use polynomials of order k for the approximation of the numerical traces, and polynomials of order k+1 for that of scalar variables and their gradients. We show the schemes are stable and uniquely solvable. By employing reduced stabilization, we establish optimal convergence rates for all variables in the L2 norm, which is valid for polynomials of arbitrary order. In terms of globally coupled degrees of freedom, the scheme is superconvergent.

Deep learning-based method for solve incompressible Navier-Stokes equation

HE Qiaolin
Sichuan University

We extend the algorithm presented by Han, Jentzen and E to Navier-Stokes in high dimension, which is an initial boundary value problem. The equation is reformulated using backward stochastic differential equations and the gradient of the unknown solution is approximated by neural networks. Numerical examples show the accuracy of the algorithm, which is quite effective in high dimension.

Sharp-interface approaches for simulating solid-state dewetting problems

JIANG Wei
Wuhan University

Thin Solid films are usually thermodynamically unstable in the as-deposited state. Heating can lead to fragmentation of a thin film and the formation of micro-/nano- solid particles, which is called as solid-state dewetting. This process is often driven by the minimization of the total interfacial energy of the system. In this talk, I will talk about how to use sharp-interface models for simulating solid-state dewetting problems. Taking the 2D case for example, I will explain the main idea behind these approaches, which includes how to derive mathematical models and how to use parametric finite element methods to numerically solve these models. Some extensions to the 3D case will be also presented.

Modeling and nonlinear simulation of solid tumor growth with chemotaxis

LI Shuwang
Illinois Institute of Technology

We develop a model for simulating the nonlinear dynamics of a tumor-host interface within the sharp interface framework. We consider solid tumor growth with chemotaxis and cell-to-cell adhesion, together with the effect of the tumor microenvironment by the variability in spatial diffusion gradients, the uptake rate of nutrients inside/outside the tumor and the heterogeneous distribution of vasculature modeled using complex far-field geometries. We solve the nutrient field (modified Helmholtz equation) and the Stokes/Darcy flow field using a spectrally accurate boundary integral method, and update the interface using a nonstiff semi-implicit approach. Numerical results highlight the complexity of the problem, e.g. development of spreading branching-patterns and encapsulated morphologies in a long period of time.

Thermodynamically consistent phase-field modelling and its application in moving contact line problems and vesicle motions

LIN Ping
University of Dundee

We will first show how to develop a thermodynamically consistent phase field model for the binary incompressible (quasi-incompressible) fluid with thermocapillary effects, which allows for the different properties (densities, viscosities and heat conductivities) of each fluid component. A sharp-interface limit analysis is carried out to show that the interfacial conditions of the classical sharp-interface conditions can be recovered or developed. We then apply the modelling method to derive a model for variable density moving contact line problems. Energy law preserving computational methods are developed for variable density models. A number of illustrative computational examples will be presented. At the end we will also show how to apply the modelling method to vesicle motions through a narrowed channel. This talk is based on a few joint papers with Zhenlin Guo, Lingyue Shen, Shixin Xu, Zhiliang Xu and Huaxiong Huang.

Boundary Effects in General Diffusion

LIU Chun
Illinois Institute of Technology

In this talk we will discuss several dynamic boundary conditions for systems of general diffusions. The models could be derived from the energetic variational framework, with the suitable kinematic constraints.

Simulation of electro-wetting on dielectric

REN Weiqing
National University of Singapore

We consider a charged droplet on a dielectric substrate and study the interface near the contact line. We first consider the static problem. We derive the governing equations using the principle of minimum energy and analyse the behaviour of the apparent contact angle in the limit when both the Maxwell stress and the thickness of the dielectric substrate go to zero. We then consider the dynamical problem and propose a finite element method for the simulation of the dynamics. Numerical examples will be presented to demonstrate the efficiency of the numerical method.

Highly efficient and accurate positivity preserving and energy stable schemes for second- and fourth-order nonlinear systems

SHEN Jie
Purdue University

Many complex nonlinear systems have intrinsic structures such as energy dissipation and positivity/maximum principle preserving. It is desirable, sometimes necessary, to preserve these structures in a numerical scheme.

I will present a procedure based on the SAV approach with transformation, to construct efficient energy stable and positivity preserving schemes for a class of second- and fourth-order nonlinear evolution systems whose solutions remain to be positive or in a fixed range.

Modeling Multi-Component Two-Phase Systems with Partial Miscibility using Diffuse Interface Models Based on Equations of State

SUN Shuyu
King Abdullah University of Science & Technology

Two or multiple phases in fluid mixture commonly occur in petroleum industry, where oil, gas and water are often produced and transported together. Petroleum reservoir engineers spent great efforts in drainage problems arising from the development and production of oil and gas reservoirs so as to obtain a high economic recovery, by developing, conducting, and interpolating the simulation of subsurface flows of reservoir fluids, including water, hydrocarbon, CO2, H2S for example in porous geological formation. Field-scale or Darcy-scale simulation has conventionally and routinely used for this purpose, but this approach requires a number of poorly known parameters including relative permeability and capillary pressure. Pore-scale simulation of two-phase systems, however, can lead to information on these parameters as well as to provide deep understanding of porous media flow and transport phenomena.

In this work, we consider two-phase flow with partial miscibility at a pore scale. Specifically, we study the modeling and simulation of possibly compressible, partially miscible, fully compositional two-phase hydrocarbon systems using diffuse interface models together with the Peng-Robinson Equation of State (PR-EoS). This research has an eventual goal of applying to realistic modeling of petroleum and other reservoir fluids in pores or pore networks within geological formation. Our modeling scheme utilizes molar densities as the order parameters, and the approach is based on the coupling of the Navier-Stokes equation for flow and a Cahn-Hilliard-like equation with Peng-Robinson chemical potentials for phase behaviors of hydrocarbon fluids. Our modeling approach can be used to predict volumetric behaviors, solubility, miscibility, and interface tensions of common hydrocarbon liquid (oil) and vapor (gas) accurately. Moreover, the entire modeling approach is self-

consistent and complies with the principles of non-equilibrium thermodynamics including the second law of thermodynamics and the Onsager reciprocity principle. To solve the continuum model expressed as a coupled nonlinear partial differential equation (PDE) system, we propose an efficient numerical solution of the modeling system, focusing on discrete energy stability, local mass conservation and numerical accuracy. For spatial discretization, we apply a finite volume-based method to turn the partial differential equations (PDE) into an ordinary differential equation (ODE) system. For temporal discretization, the resultant ODE system is decoupled by using an asymmetric splitting scheme, and then integrated in time using a semi-implicit marching scheme. In addition, targeting the specific features of each of the three terms in Peng-Robinson chemical potentials, we propose a number of energy-stable semi-implicit time schemes. Some of our methods are based on convex-concave splitting of Peng-Robinson Equation of State (PR-EoS), and some other methods of ours were constructed based on the scalar auxiliary variable (SAV) approaches for Helmholtz free energy and proper intermediate velocities were designed to decouple the tight relationship between velocity and molar densities. In this talk, I will review those methods, and emphasize our most recent approach based on novel energy factorization (EF), which first factorizes an energy function into a product of several factors and then treats the factors using their properties to obtain the semi-implicit linear schemes. In this method, we apply the EF approach to deal with the Helmholtz free energy density determined by PR-EoS, and then propose a linear semi-implicit numerical scheme that inherits the original energy dissipation law. Numerical results are presented to demonstrate the stability and efficiency of the proposed scheme. We compare our computational results with laboratory experimental data reported in the literature, which have good agreement.

This presentation is partially based on joint work with Jisheng Kou (Hubei Eng. U.), Yiteng Li (KAUST), Zhonghua Qiao (HK PolyU), and Tao Zhang (KAUST).

An efficient iterative convolution thresholding method (ICTM) for free interface related optimization problems

WANG Dong
The Chinese University of Hong Kong, Shenzhen

In this talk, we will present an efficient iterative convolution-thresholding method (ICTM) for solving interface related optimization problems. The approximation, derivation and stability analysis of the method will be presented. The method is shown to be unconditionally stable, efficient, simple, easy to code, and applicable to a wide range of problems. Applications in image processing, surface reconstruction, topology optimization, and biological networks will be presented.

Thermodynamically consistent algorithms for incompressible diblock copolymer solutions coupled with electric and magnetic fields

WANG Qi
University of South Carolina

We derive thermodynamically consistent models for diblock copolymer solutions coupled with the electric and magnetic field, respectively. These models satisfy the second law of thermodynamics and therefore are therefore thermodynamically consistent. We then design a set of 2nd order, linear, semi-discrete schemes for the models using the energy quadratization method and the supplementary variable method, respectively, which preserve energy dissipation rates of the models. The spatial discretization is carried out subsequently using 2nd order finite difference methods, leading to fully discrete algorithms that preserve discrete energy12 dissipation-rates of the models so that the resulting fully discrete models are thermodynamically consistent. Convergence rates are numerically confirmed through mesh refinement tests and several numerical examples are given to demonstrate the role of the mobility in pattern formation, defect removing effect of both electric and magnetic fields as well as the hysteresis effect for applied external fields in copolymer solutions.

On variational Lagrangian methods for phase-field models and beyond: A discrete energetic variational approach

WANG Yiwei
Illinois Institute of Technology

In this talk, we present a systematic framework of deriving variational numerical methods for generalized diffusions and gradient flows. The numerical framework is based on the energy-dissipation law, which describes all the physics and the assumptions in a given system, and can combine different types of spatial discretizations including Eulerian, Lagrangian, and particle-based approaches. The resulting semi-discrete equation inherits the variational structures from the continuous energy-dissipation law. As examples, we apply such an approach to construct variational Lagrangian schemes to the Allen-Cahn type phase-field models and the porous medium type generalized diffusions. Numerical examples show the advantages of the variational Lagrangian schemes in capturing singularities, thin diffuse interfaces, and free boundaries. This is joint work with Professor Chun Liu.

Modeling growth and morphogenesis in biological tissues

WEI Chaozhen
Worcester Polytechnic Institute

Understanding the mechanics of growth and morphogenesis in living tissues has been a subject of intense interest but remains an open problem. Mathematical modeling of this problem is challenging due to the coupling between mechanical stress and active bioactivities. We develop a generic continuum mathematical framework for describing the growth of soft compressible tissues, accounting for the external diffusible growth factor field and the mechanical feedback mechanism. We explicitly describe the active tissue rearrangement by introducing the adaptive reference map that consequently induces to the Maxwellian stress relaxation of viscoelastic material. We employ the model to probe the roles of mechanical properties in tissue growth.

Modeling Effects of Randomness on the Strength of High-Entropy Alloys

XIANG Yang
The Hong Kong University of Science and Technology

High-entropy alloys (HEAs), i.e., single-phase, nearly equiatomic multicomponent, metallic materials, are associated with novel mechanical properties, such as high strength, fracture resistance etc. We propose a stochastic Peierls-Nabarro (PN) model to understand how random site occupancy affects intrinsic strength. We also present an asymptotic derivation of the stochastic continuum model from an atomistic model that incorporates the atomic level randomness. The model predicts the intrinsic strength of HEAs as a function of the standard deviation and the correlation length of the randomness. This approach provides a fundamental explanation to the origin of the high strength of HEAs.

An Energy Stable C^0 Finite Element Scheme for A Phase-Field Model of Vesicle Motion and Deformation

XU Shixin
Duke Kunshan University

A thermodynamically consistent phase-field model is introduced for simulating motion and shape transformation of vesicles under flow conditions. In particular, a general slip boundary condition is used to describe the interaction between vesicles and the wall of the fluid domain. A second-order accurate in both space and time C^0 finite element method is proposed to solve the model governing equations. Various numerical tests confirm the convergence, energy stability, and conservation of mass and surface area of cells of the proposed scheme. Vesicles with different mechanical properties are also used to explain the pathological risk for patients with sickle cell disease.

A diffusion-generated method for wetting on inhomogeneous surfaces

XU Xianmin
Chinese Academy of Sciences

Wetting on rough surfaces is an interesting phenomenon in nature and has many applications in some industrial processes. The microscopic roughness and chemically inhomogeneity cause many difficulties in analysis and simulations for this problem. By using the Onsager variational principle as an approximation tool, we develop a new diffusion generated motion method for wetting problems. The method uses a signed distance function to represent the interface between the liquid and vapor surface. In each iteration, a linear diffusion equation with a linear boundary condition is solved for one time step in addition to a simple re-distance step and a volume correction step. Its energy stability property is analysed by careful studies for some geometric flows on substrates. Numerical examples show that the method has optimal convergence order and can be used to simulate complicated wetting problems on inhomogeneous surfaces.

Onsager’s variational principles in active soft matter

XU Xinpeng
Guangdong Technion-Israel Institute of Technology

Onsager’s variational principle (OVP) was originally proposed by Lars Onsager in 1931 [L. Onsager, Phys. Rev., 1931, 37, 405]. This fundamental principle provides a very powerful tool for formulating thermodynamically consistent models in nonequilibrium thermodynamics. It can also be employed to find approximate solutions, especially in the study of soft matter dynamics. In this talk, I will show that OVP can be extended and applied to the dynamic modeling of active soft matter such as suspensions of bacteria and aggregates of animal cells. I will start with a brief introduction to the biology and artificial systems that motivate the studies of active soft matter. I will then quickly review the classical OVP and its extension to active matter dynamics where active forces are included as

external non-conservative forces. Next, I use OVP to analyze the directional "walk" of an individual molecular motor on a stiff biofilament. After that I will use OVP to formulate a diffuse-interface model for an active polar droplet on a solid substrate. In addition to the generalized hydrodynamic equations for active polar fluids in the bulk region, we have also derived thermodynamically consistent boundary conditions. Particularly, when the active droplet is very thin, the lubrication approximation can be applied. In this case, I will show that a generalized thin film equation can be derived using OVP. Moreover, OVP can also be used as an approximation tool to find the spreading laws for the thin active polar droplet. Finally, I use OVP to analyze the directional locomotion of a toy two-sphere microswimmer, in which nonlinear dissipation function arises that proposes significant challenges to OVP. In summary, the variational method we have proposed by incorporating biochemical activity into OVP will help to deepen our understanding of the emergent structure and dynamic behaviors of real in vivo biological systems such as bacteria suspensions, individual animal cells and cell aggregates (or tissues).

A cell-centered finite volume method on locally refined composite Cartesian grids and its application in wetting problems

YING Wenjun
Shanghai Jiaotong University

In this talk, I will present a cell-centered finite volume method on locally uniformly refined composite Cartesian grids. Unlike similar methods on composite Cartesian grids (such as those proposed by McCormick Thomas 1986, Bramble-Ewing-Pasciak-Schatz 1988, Johansen-Colella 1998, Papac-HelgadottirRatsch-Gibou 2013 and Kriva-Handlovicova 2016), the method is derived in a very simple way based on finite volume conservation of mass and flux. The finite volume stencils on composite grids are compact in both two and three space dimensions. I will also describe an efficient multilevel/multigrid (composite grid) iteration technique for the Poisson equation with the cell-centered finite method as well as its application in a three-dimensional wetting problem for moving contact lines. This is joint work with Xianmin Xu (CAS) and Zhongshu Zhao (SJTU).

Phase-field modeling of contact angle hysteresis and its application in drop impact dynamics

YUE Pengtao
Virginia Polytechnic Institute and State University

In reality, most solid surfaces are intrinsically rough or chemically heterogeneous. As a consequence, the contact line may get pinned at topological or chemical defects, which is known as contact angle hysteresis. In this talk, we introduce a thermodynamically consistent phase-field model that accurately captures contact angle hysteresis. The formulation satisfies a dissipative energy law, based on which we develop a finite-element discretization that is unconditionally energy-stable in some simple cases. In the end, we use this method to simulate the transient process of a drop impacting a solid surface, where the drop may deposit, rebound, or partially rebound depending on the drop impact velocity and the surface wetting property. By tuning the hysteresis window, our numerical results can achieve very good agreement with the experimental measurements, which also manifests the importance of hysteresis in contact line dynamics.

Dynamics of Grain Boundaries with Microscopic Constraints

ZHANG Luchan
The Hong Kong University of Science and Technology

Dynamic properties of grain boundaries play vital roles in the mechanical and plastic behaviors of polycrystalline materials. The properties of grain boundaries strongly depend on their microscopic structures. We present continuum models for the dynamics of grain boundaries based on the continuum distribution of the line defects (dislocations or disconnections) and the constraints associated them.

The long-range elastic interaction between the line defects is included in the continuum models to maintain stable microstructure on grain boundaries during the evolution. However, the calculation of the long-range force is quite time-consuming due to its form of integrals over all the grain boundaries. This limitation can be addressed by replacing with constraints that governs the stable dislocation structure during the evolution. In the other hand, in polycrystalline materials, the motion of grain boundaries is inevitably constrained by other grains and triple junctions, and this essentially influences the materials properties. We incorporate these important microscopic constraints to continuum dynamics models of high-angle grain boundaries whose dynamics are controlled by motion of disconnections.

Fourth-order projection methods for incompressible Navier-Stokes equations on irregular no-slip domains

ZHANG Qinghai
Zhejiang University

Previously we have developed a fourth-order finite-volume method for numerical solving the incompressible Navier-Stokes equations (INSE) on rectangular domains. We have also established a topological space for modeling physically meaningful irregular regions. In this talk we present our recent effort on coupling the previous two modules to solve the INSE on irregular no-slip domains. Numerical results show fourth-order convergence both in time and in space.

A finite element method for Electrowetting on Dielectric

ZHAO Quan
National University of Singapore

We consider the problem of electrowetting on dielectric (EWoD). The system involves the dynamics of a conducting droplet, which is immersed in another dielectric fluid, on a dielectric substrate under an applied voltage. The fluid dynamics is modeled by the two-phase incompressible Navier-Stokes equations with the standard interface conditions, the Navier slip condition on the substrate, and a contact angle condition which relates the dynamic contact angle and the contact line velocity, as well as the kinematic condition for the evolution of the interface. The electric force acting on the fluid interface is modeled by Maxwell's equations in the domain occupied by the dielectric fluid and the dielectric substrate. We develop a numerical method for the model based on its weak form. This method combines the finite element method for the Navier-Stokes equations on a fixed bulk mesh with a parametric finite element method for the dynamics of the fluid interface and the boundary integral method for the electric force along with the fluid interface. Numerical examples are presented to demonstrate the accuracy and convergence of the numerical method, the effect of various physical parameters on the interface profile, and other interesting phenomena such as the transportation of droplet driven by the applied non-uniform electric potential difference.

A high-order fictitious-domain method for linear convection-diffusion equation on time-varing domain

ZHENG Weiying
Chinese Academy of Sciences

We develop a high-order finite element method to solve the linear convection-diffusion equation on a time-varying domain. The method is based on a characteristic-Galerkin formulation combined with the BDF-k and the fictitious-domain finite element method. Optimal error estimates of the discrete solutions are proven for $2 \leq k \leq 5$ by taking account of the errors from surface-tracking, temporal discretization, and spatial discretization, provided that the (k+1)th-order Runge-Kutta scheme is used for surface-tracking. Numerical experiments demonstrate the optimal convergence of the method for k=3 and 4.